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EXPERIMENTAL AND NUMERICAL INVESTIGATIONS OF THE GAUSSIAN SUPPRESSION OF SOUND BY SOUND

by

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EXPERIMENTAL AND NUMERICAL INVESTIGATIONS OF THE GAUSSIAN SUPPRESSION OF SOUND BY SOUND

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ABSTRACT

In this work, we report on experimental and numerical investigations of the attenuation of a small-amplitude signal due to its interaction with high intensity, band limited sound whose spectrum consists of one to four discrete peaks. We probe the "thermodynamic limit" for different relative frequencies of the spectral components. In particular the attenuation of the signal is investigated for both equally and unequally spaced spectral components, as well as different phase relations among them. The possibility of collective modes is also explored by measurements of the phase change in the signal downstream due to the presence of discrete noise.

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I. INTRODUCTION

The essential process in any system of finite-amplitude noncoherent waves is the irreversible redistribution of energy among different frequencies. This gives rise to a variety of interesting and potentially useful phenomena, some of which have only recently begun to be fully explored. One such effect is the absorption of a signal by nonlinear shockless noise.

The basic nonlinear interaction of acoustic waves is a three-wave resonance, so that waves with frequency f_1 and f_2 scatter to produce waves with frequencies $f_3 = |f_1 \pm f_2|$. Furthermore, because these waves are nondispersive, resonant interactions are collinear (Rudenko and Soluyan, 1977). For random waves in two and three dimensions, energy in the noise components redistributes irreversibly by collinear resonant interactions. After a time determined by the nonlinearities, a closed system of interacting random waves can reach thermodynamic equilibrium, and small fluctuations about this state relax exponentially with an attenuation coefficient proportional to the noise intensity. A small-amplitude wave that is injected into this noise can be regarded as a fluctuation of the noise. Whereas the attenuation of this signal is predicted (Westervelt, 1976) and observed (Stanton and Beyer, 1978 and 1981) to be an exponential for noise in three dimensions, in one dimension the signal is predicted (Rudenko and Chirkin, 1975; Rudenko and Soluyan, 1977) to attenuate as a Gaussian in the distance from the source. Experimental verification of the Gaussian attenuation has been recently reported (Larraza et al., 1996).

The Gaussian attenuation in one-dimensional nonlinear acoustics brings together two physically significant results. First, because this fluctuation does not relax exponentially, we infer that a system of interacting, collinear,

nondispersive waves is inherently far off equilibrium. This results from both a lack of angle randomization and the fact that all spectral components are in resonance with each other in this case. A second aspect of a Gaussian attenuation is that it breaks translational invariance because the attenuation coefficient is now a function of position. The system therefore has memory. Specifically, by taking measurements at several distances from the source of a signal, one can calculate both the location and strength of the source. This remote detection cannot be achieved if the attenuation is exponential. This is physically significant because the equations that describe the evolution of nonlinear noise and its interaction with a monofrequency signal are translationally invariant.

Because one-dimensional acoustic noise is inherently far off equilibrium, it belongs to the general class of systems that exhibit wave turbulence (Zakharov et al., 1992). Collective modes, in which energy fluctuations propagate, have been predicted in certain far off equilibrium dispersive systems (Larraza and Falkovich, 1993), but there are currently no controlled observations. These modes are important, for example, as a mechanism of energy transfer. In contrast to other wave turbulent systems, nonlinear acoustic noise in one dimension is unique in that it lacks a Boltzmann-type description of interacting waves. The existence of collective modes in one-dimensional acoustic noise is thus unknown. Larraza et al. (1996) have established a first step in this possibility: the first observation of the ω^{-3} power law spectrum that is predicted (Kuznetsov, 1970) to occur for fully-developed shockless noise.

Having established some of the properties of broadband noise, a natural path of research is to investigate the effect on a small-amplitude signal of band-limited noise whose spectrum consists of discrete peaks. In particular, it is natural to ask how many peaks would lead to the same absorption of a signal as

a continuous band of noise. By reducing the number of peaks while keeping the overall intensity constant, we can investigate the interesting fundamental problem of how many degrees of freedom are required for the system to behave "thermodynamically" (such that the attenuation is Gaussian).

In this thesis, we report on experimental and numerical investigations of the attenuation of a small-amplitude signal due to its interaction with high intensity, band-limited sound whose spectrum consists of one to four discrete peaks. We probe the "thermodynamic limit" for different relative frequencies of the spectral components. In particular, the attenuation of the signal is investigated for both equally and unequally spaced spectral components, as well as different phase relations among them. The possibility of collective modes is also explored by measurements of the phase change in the signal downstream due to the presence of discrete noise.

A general theoretical background is presented in Ch. II, while Ch. III deals with the results of numerical investigations in the absence of viscous and thermal losses. The results of the experiments are presented in Ch. IV. In Ch. V we present the conclusions derived from the research, as well as recommendations for further work.

II. THEORY

The propagation of sound in one dimension in a barotropic fluid is described by the particularly simple equation (Blackstock, 1972)

$$u_t + (c + \beta u)u_x = 0 \ , \tag{2.1}$$

known as the Riemann equation. Equation (2.1) describes the propagation of waves in one direction (toward increasing x), were $u(x,t)$ is the fluid particle velocity and $c^2 = (\partial p/\partial \rho)_0$ is the equilibrium value of the square speed of sound. For an ideal gas, the nonlinear coefficient is $\beta = (1+\gamma)/2$, where γ is the ratio of specific heats of the gas. From Eq. (2.1), the instantaneous velocity of propagation of a point in the wave with particle velocity u is $c+\beta u$. Thus, points of the wave having different particle velocity propagate at different speeds.

A general implicit solution of Eq. (2.1) can be achieved if we note that a point of constant particle velocity u of the wave moves with velocity $c+\beta u$. The particle velocity at (x,t) is thus equal to the particle velocity at $(0,\tau)$ where the retarded time is

$$\tau = t - \frac{x}{c + \beta u} \ . \tag{2.2}$$

A general implicit solution of Eq. (2.1) with the $x = 0$ boundary condition $u(0,t) = f(t)$ is thus

$$u(x,t) = f\left(t - \frac{x}{c + \beta u(x,t)}\right) . \tag{2.3}$$

The velocity at the boundary $x = 0$ thus determines the velocity at $x > 0$ through the retarded time.

The simplicity of Riemann's solution allows the determination of exact and asymptotic explicit solutions in the preshock region. Examples include Fubini's (1935) pure tone radiation problem, Fenlon's (1970) generalization to multiple frequency sinusoidal source, Kutznetsov's (1970) asymptotic ω^{-3} spectrum for broadband noise, and Rudenko and Chirkin's (1975) Gaussian attenuation of a signal due to interactions with normally distributed random noise.

The main theme in this thesis is to show that such a simple equation describes a rich number of fundamental phenomena. In this chapter, we present the analytic solutions to some special cases. Experimentally, the dimensionality is controlled by performing measurements in a long traveling wave tube driven at frequencies below the first cutoff. Thus, the effects of wall losses become important. We show how to account for wall losses and give the modifications to the dissipation–free exact solutions.

A. SHOCK INCEPTION DISTANCE

At $x = 0$, suppose particle velocity u_1 occurs at time t_1, and a greater particle velocity u_2 occurs at a later time t_2. Riemann's equation (2.1) is invalid when the second disturbance overtakes the first, which will eventually occur at some distance $x > 0$. This overtaking yields a discontinuity (infinite slope or shock) in u. If t_1 and t_2 differ by a finite amount, a discontinuity will first occur at a distance *less* than x, because the waveform $u(x,t)$ continuously distorts. We are thus led to consider velocity disturbances u and u+du and at $x = 0$ that are an infinitesimal time dt apart. If $du > 0$, the disturbances will coincide at some

distance L, after which Riemann's equation is invalid. If T is the elapsed time, then

$$[c + \beta u]T = L \ , \tag{2.4a}$$

$$[c + \beta(u + du)](T - dt) = L \ , \tag{2.4b}$$

corresponding to the first and second disturbances, respectively. Multiplying out Eq. (2.4b), substituting Eq. (2.4a), and solving for L, yields

$$L = \frac{(c+\beta f)^2}{\beta\, df/dt} \ . \tag{2.5}$$

The quantities f and df/dt are the values of the particle velocity and its slope at x = 0, respectively. The expression (2.5) gives the distance at which an initial positive slope becomes infinite. (A negative slope never becomes infinite.) For any solution to Riemann's equation to be valid at some distance x, it must be verified that x < L for all values of f(t).

For the propagation of a pure tone, the boundary condition at x = 0 may be written as

$$u(0) = u_0 \sin\omega t \ . \tag{2.6}$$

The minimum value of the shock length (2.5) corresponds to ωt being an integral multiple of 2π. The minimum distance at which shocking occurs for a pure tone is thus

$$L = \frac{c^2}{\beta u_0 \omega} . \tag{2.7}$$

B. SUPPRESSION OF A SIGNAL BY A PUMP

The simplest problem of interaction of sound with sound is given by the x = 0 boundary condition

$$u(0,t) = u_p \sin(\omega_p t) + u_s \sin(\omega_s t) , \tag{2.8}$$

where we focus here on the interaction of a strong pump wave of low frequency ω_p with a weak signal of high frequency ω_s. The signal has negligible effect on the propagation of the pump. However, the pump will modulate the signal and generate sidebands at frequencies $\omega_s \pm \omega_p$, thereby removing energy from the signal. An approximation to Fenlon's (1970) solution for the evolution of the signal is

$$u(x,t) = u_s J_o\left(\beta u_p \omega_s x / c^2\right) \sin[\omega_s (t - x/c)] . \tag{2.9}$$

The amplitude of the signal depends only on the particle velocity of the pump, the frequency of the signal, and the distance. The signal vanishes at the zeros of the Bessel function, at which points the energy has been temporarily completely pumped into adjacent sidebands.

C. ABSORPTION BY NOISE

For the case of a weak signal of frequency f_0 in the presence of finite-amplitude broadband noise, the amplitude of the signal attenuates as a Gaussian $\exp(-\Gamma x^2)$, where

$$\Gamma = \frac{2\pi^2\beta^2 f_0^2}{c^4} u_{rms}^2 , \qquad (2.10)$$

where u_{rms} is the rms velocity of the noise (Rudenko and Chirkin, 1975; Rudenko and Soluyan, 1977). Broadband noise may be considered by the $x = 0$ boundary condition

$$u_{noise}(0,t) = B\sum_{n=1}^{N}\cos(\omega_n t + \varphi_n) , \qquad (2.11)$$

where B is the peak velocity of each component of the noise, ω_n are densely distributed frequencies, and φ_n are randomly distributed phases. The Gaussian attenuation thus results as a limiting case of multiple pump waves with random phases. When a signal is interacting with one pump, there are regions of space where restitution occurs (amplitude increases with distance). This effect is lost when the interaction occurs with a large number of pumps with random phases. Because of the central limit theorem, the noise described by Eq. (2.11) should approach a normal distributed noise for a sufficiently large number N. In this limit, a theory formulated in terms of the noise (2.12) should accurately yield the Gaussian attenuation.

By squaring Eq. (2.11) and averaging over time, we determine the square rms particle velocity of the noise. Solving for B yields

$$B = u_{rms}\sqrt{\frac{2}{N}} \, . \tag{2.12}$$

The approach to the Gaussian attenuation can be probed by increasing N while keeping u_{rms} constant.

D. EFFECTS OF WALL LOSSES

In the absence of wall losses, a monofrequency weak signal interacting with noise attenuates as a Gaussian with attenuation coefficient (2.10). The amplitude therefore evolves according to

$$\frac{dA}{dx} = -\frac{4\pi^2\beta^2 f_0^2}{\rho c^5} IxA \, , \tag{2.13}$$

where f_0 is the frequency of the signal and $I = \rho c u_{rms}^2$ is the intensity of the noise.

Due to thermal and viscous losses, the intensity of the noise does not remain constant as a function of position. Specifically, each spectral energy component of the noise evolves according to

$$I_f(x) = I_f(0)\,\exp(-2\alpha\sqrt{f}\,x) \, , \tag{2.14}$$

where $I_f(0)$ is the spectral shape at the origin and $\alpha\sqrt{f}$ is the linear attenuation coefficient (Kinsler et al., 1982). Eq. (2.14) neglects nonlinear losses due to energy transfer to other frequencies, and assumes that the dominant mechanisms are thermoviscous losses at the wall. In the shockless regime this assumption is very reasonable because, due to nonlinearities, fully developed

noise is predicted (Kuznetsov, 1970) to have an f^{-3} high frequency tail. Thus, if the noise is originally within a band of frequencies $\Delta f = f_2 - f_1$, as is the case in the experiment by Larraza et al. (1996), nearly all of the energy remains in this band.

Incorporating thermal and viscous losses at the wall, we generalize Eq. (2.13) to

$$\frac{dA}{dx} = -\frac{4\pi^2\beta^2 f_0^2}{\rho c^5} IxA - \alpha\sqrt{f_0}\,A \,, \tag{2.15}$$

where I is the total intensity of the noise. Eq. (2.15) is justified by a multiple length-scale approach where variations of the amplitude A occur over a shorter length scale than that of the noise intensity I, which holds in the experiment of Larraza et al. (1996) because the frequency of the signal is substantially greater than the upper frequency of the noise. The first term in Eq. (2.15) represents losses due to the presence of noise at location x, and the second term corresponds to thermal and viscous losses at the walls. Solving Eq. (2.15) for the signal amplitude yields

$$A = A_0 \exp\left[-\frac{4\pi^2\beta^2 f_0^2}{\rho c^5} \int_0^x I(x')x'dx'\right] e^{-\alpha\sqrt{f_0}\,x} \,, \tag{2.16}$$

which was successfully verified experimentally by Larraza et al.

In the case of interaction with a single pump in the absence of dissipation, Eq. (2.9) implies that the amplitude of the signal evolves according to the Bessel equation

$$\frac{d^2A}{dx^2}+\frac{1}{x}\frac{dA}{dx}+\left(\frac{\beta u_p \omega}{c^2}\right)^2 A=0\,, \tag{2.17}$$

with the boundary condition $A = u_s$ at $x=0$.

Due to thermal and viscous losses, the amplitude of the pump does not remain constant as a function of position but varies according to

$$u_p(x)=u_p(0)\,\exp(-\alpha\sqrt{f_p}\,x)\ , \tag{2.18}$$

where $u_p(0)$ is the amplitude of the pump at the origin. In Eq. (2.18) we have again ignored losses due to energy transfer to other higher frequencies and assumed that the dominant mechanisms are thermoviscous losses at the wall. In order to find the effects of wall losses, we can proceed in a fashion similar to the case of broadband noise, and assume that except for a pre–factor $\exp(-\alpha\sqrt{f_s})$ the amplitude of the signal evolves according to Eq. (2.17), with u_p given by Eq. (2.18). Such an approximation will hold if the length scale of amplitude variations of the signal is predominantly dictated by $c^2/\beta u_p \omega_s$, and if the length scale $1/\alpha\sqrt{f_p}$ of amplitude variations of the pump is greater than this value. The solution to this new equation with the boundary conditions $A = u_s$ and $dA/dx=0$ at $x=0$ can be obtained numerically. In Chapter IV we compare the amplitude predicted by the theory with the measured values of a weak signal interacting with a large amplitude pump.

III. NUMERICAL SIMULATION

In this chapter, we present the results of numerical integration of a signal in the presence of few components of the "noise." We briefly describe the method used to integrate Riemann's equation for sound propagation in one dimension in the shockless regime. We specialize to the cases of one, two, three, and four noise components with equal and unequal frequency spacing.

A. FORWARD PROPAGATION METHOD

As a numerical convenience, we choose $\beta = 1$ and $c = 1$ so that the dimensionless Riemann's equation (2.1) and the expression (2.2) for the retarded time become

$$\frac{\partial u}{\partial t} + (1+u)\frac{\partial u}{\partial x} = 0, \tag{3.1}$$

and

$$\tau = t - \frac{x}{1+u}, \tag{3.2}$$

respectively. As explained in Chapter II, a general implicit solution of Eq. (3.1) with the $x = 0$ boundary condition $u(0,t) = f(t)$ is given by $u(x,t) = f(\tau)$.

Eq. (3.2) determines a set of straight lines in the x-τ plane. The slope of each line is determined by the value of the velocity at the boundary at a given time (wavelet). These lines are known as *characteristics*, where the parameter τ is the time base for the source signal with $t = \tau$ at $x = 0$.

The principle of the forward propagation method stems from the fact that the characteristics represent the trajectories of motion of the different wavelets in the x-τ plane. The velocity of propagation of a wavelet is equal to 1+u where u is the assigned value of the velocity of the wavelet at the boundary. In the propagation of the initial waveform the first incremental distance produces a new time associated with the velocity. While at the origin the spacing between time steps is commensurate, once the waves propagate a small distance the time between adjacent elements is stretched or compressed according to Eq. (3.2), which physically corresponds to distortions of the initial waveform. Appendixes A and B show a C-language numerical program that implements the forward propagation method.

B. AVERAGING TECHNIQUE

Our main emphasis is in determining the evolution of the amplitude of a monofrequency signal in the presence of noise. As we forward-propagate the boundary condition of a time series with the signal plus noise, sidebands of the noise appear about the signal's frequency. Also, the amplitude of the signal is expected to attenuate with distance. It is thus necessary to extract the signal from the noise at a distance x from the source. We achieve this by Fourier analyzing the waveform at the frequency of the signal, which amounts to time averaging the in-phase and quadrature components of the waveform. This process is referred to as the "discrete Fourier transform" (DFT) because the integration is performed with a finite (rather than continuous) time increment and is performed over a finite (rather than infinite) time interval.

The integration in our DFT is accomplished with trapezoidal summation. (Simpson's rule cannot be employed because the time increment is not

constant.) Appendixes A and B show the numerical program which includes the trapezoidal summation.

The integration over finite rather than infinite time intervals causes errors in the signal amplitude output. For equally spaced noise components, this error can be eliminated completely by averaging over an integral number of cycles n of the signal such that $n = f/\Delta f$, where f is the frequency of the signal and Δf is the spacing between spectral components of the noise (Jang, 1996). However, because the time increment is not constant n is only a lower bound.

We investigated numerically the proper averaging time (measured by the cycles of iteration) and its effect on calculated signal amplitude at the fixed position x=1.5 downstream in the system. Figure 3.1 illustrates the result of the calculated signal amplitude as a function of number of cycles when the noise is composed of four equally spaced spectral components, or pumps, between 1 and 2 frequency units. The frequency of the signal was chosen to be 50 frequency units. The relative phase among the four pumps was randomly varied. The figure shows that the amplitude of the signal reaches a constant value after 1,000,000 cycles of the signal. The final output depends on the seed value that was used to initialize the relative phases, and thus the signal amplitude output depends on the relative phases of the noise components. This result has been shown to be true for up to 50 equally spaced spectral components of the noise (Denardo et al., 1997).

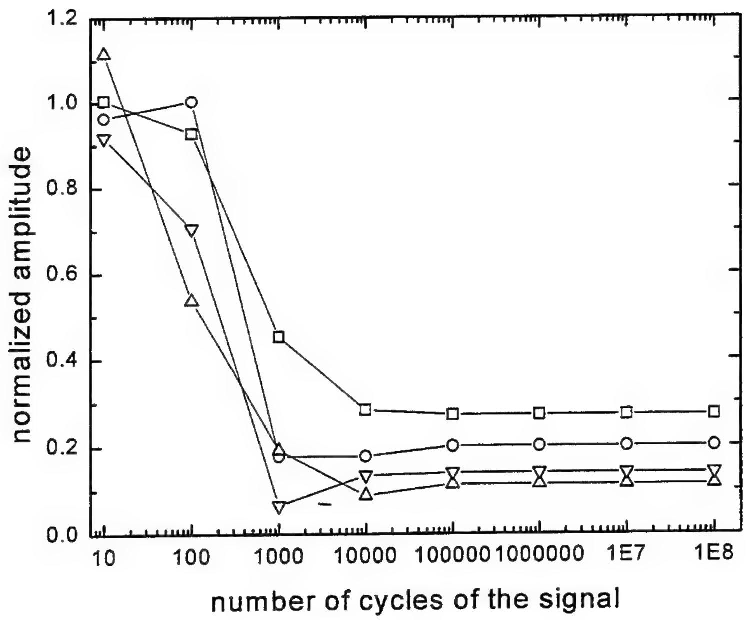

Figure 3.1 Numerical result of the nonlinear interaction of weak signal and four equally spaced noise components. The amplitude of the signal is measured at the fixed location x=1.5 downstream. The different symbols refer to four different seed values which yield four different sets of relative random phases of the noise components. The amplitude of the signal depends upon the sets of random phases.

We next investigated the effect on the signal due to unequally spaced noise components. The amplitude of the signal is also calculated at the fixed location x=1.5 downstream. The seed values were used to initialize both the relative phases of the noise components, and the unequal frequency spacing. The unequal spacing between spectral components of the noise was established by adding a random step of roughly 1% to the equally spaced noise components. The numerical computer program with the modifications is listed in Appendix A. We investigated numerically the proper averaging time in this case as well. A number of cycles greater than 1,000,000 was needed to get a stable output. However, in contrast with the equally spaced case, changing the seed value did not change the stable amplitude output, as shown in Fig 3.2. Thus, when the noise spectrum is composed of unequally spaced spectral components, the

amplitude of the signal amplitude is independent of the relative phases and frequencies of the noise components.

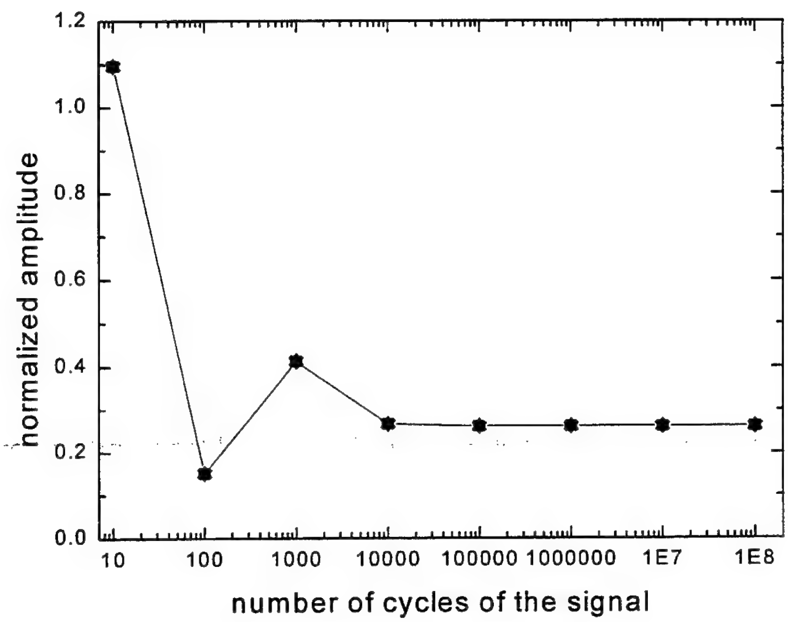

Figure 3.2 Numerical result of the nonlinear interaction of weak signal and four unequally spaced noise components. The amplitude of the signal is measured at the fixed location x=1.5 downstream. Four different seed values where used where each one initialized both the relative random phases and the amount of unequal spacing between the noise components. The amplitude of the signal is independent of the seed value.

C. INTERACTION OF A SIGNAL WITH A SMALL NUMBER OF PUMPS

The main conclusion of the previous section is that the attenuation of a weak signal due its interaction with discrete pumps that are unequally spaced in frequency is independent of the relative phase of the pumps and the value of their frequencies. In this section we investigate the spatial dependence of the amplitude for one, two, three and four pumps with unequally spaced frequencies.

As discussed in Sec. 2.2, the suppression of a signal by a pump is formulated in terms of the boundary condition (2.8) at x = 0

$$u(0,t) = u_p \sin(2\pi f_p t) + u_s \sin(2\pi f_s t) \ , \tag{3.3}$$

where the pump frequency is $f_p = 1$ and the signal frequency is $f_s = 50$. The weak signal has amplitude 10^{-4} and has little effect on the propagation of the pump with amplitude 10^{-2}. However, the pump will modulate the signal and generate sidebands at frequencies $f_s \pm f_p$, thereby removing energy from the signal. In dimensionless variables, Fenlon's approximate solution (2.9) for the evolution of the signal is

$$u(x,t) = u_s J_o(2\pi f_s u_p x) \sin[2\pi f_s (t - x/c)] \ . \tag{3.4}$$

Figure 3.3 shows a comparison between the numerical integration and the theory (3.4). The shock inception distance (2.5) for the pump equals 15.92 units. The amplitude was calculated by averaging over 10^6 cycles of the signal. There is excellent agreement between theory and numerical integration.

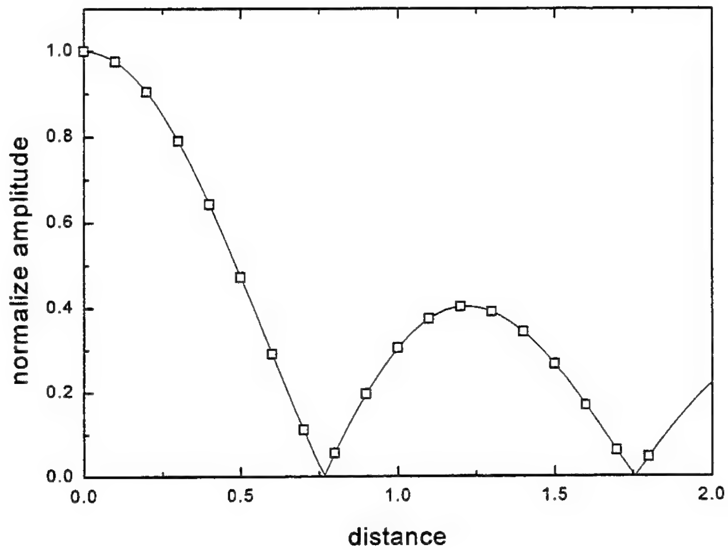

Fig 3.3 Suppression of a signal with frequency 50 by a pump of amplitude 10^{-2} and frequency 1. The signal's amplitude is normalized to its value of 10^{-4} at the origin. The points are the result of numerical integration, and the solid line corresponds to the theoretical expression (3.4). The amplitude of the signal was calculated by averaging over 10^6 cycles of the signal is in the averaging.

Unlike the case of one pump, there is no closed-form analytic expression for the amplitude of a weak signal in the presence of several pumps. Figure 3.4 shows the evolution of the amplitude of a signal as a function of distance in the presence of two pumps, whose frequencies are 1 and 1.25. Note that there is suppression of the signal in this case at a dimensionless distance of about 1.1 (compare with 0.75 for the case of a single pump). Also plotted in Figure 3.4 are both, the theory Eq. (3.4) and the Gaussian attenuation theory Eq. (2.13) for a pump with the same root mean square amplitude and for broadband noise of the same energy of the pumps respectively.

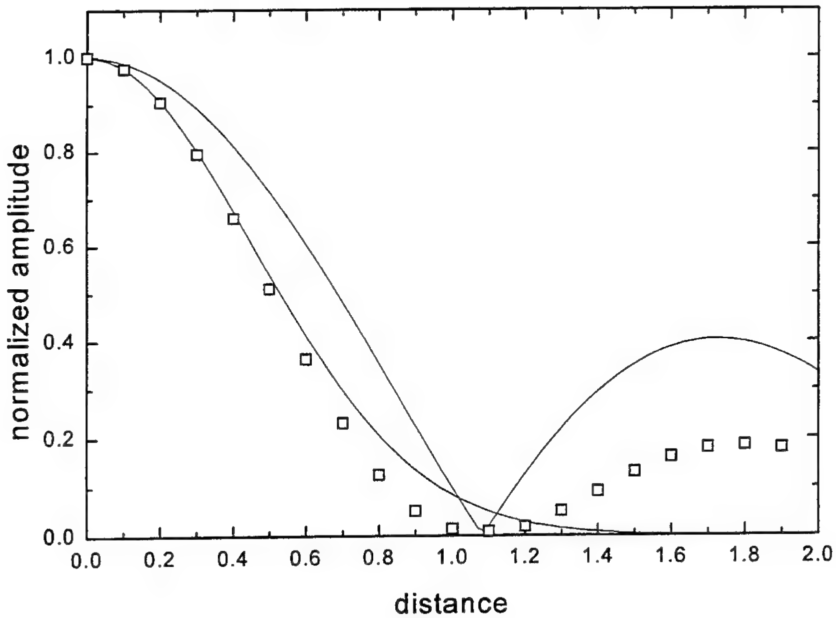

Fig 3.4 Suppression of a signal with frequency 50 by two pumps. The rms particle velocity of the pumps is 10^{-2} and the frequencies are between 1 and 2. The signal's amplitude is normalized to its value of 10^{-4} at the origin. The points are the result of numerical integration, and the solid lines correspond to the theoretical expression (3.4) and the Gaussian attenuation law (2.13). The number of cycles of the signal is 10^6.

Figure 3.5 shows the amplitude of a weak signal with frequency of 50 in the presence of three and four pumps with unequally spaced frequencies between 1 and 2. We used the values are 1, $2^{1/4}$, $2^{1/3}$ for the frequencies of the three pump, and the values 1, $2^{1/4}$, $2^{1/3}$, $2^{1/2}$ for the frequencies of the four pumps. We also plot the Gaussian attenuation theory for broadband noise with the same energy as that of the pumps. Remarkably, the attenuation of the signal due to its interaction with four pumps follow closely the Gaussian attenuation due to broad band noise. On the other hand, when the signal interacts with three pumps, the restitution effect of the signal amplitude is apparent. As we will see in Chapter IV, it is also experimentally observed that the amplitude of the signal evolves closely as a Gaussian due to its interaction with four pumps, and the

departures from Gaussian behavior are more pronounced when the signal interacts with three pumps.

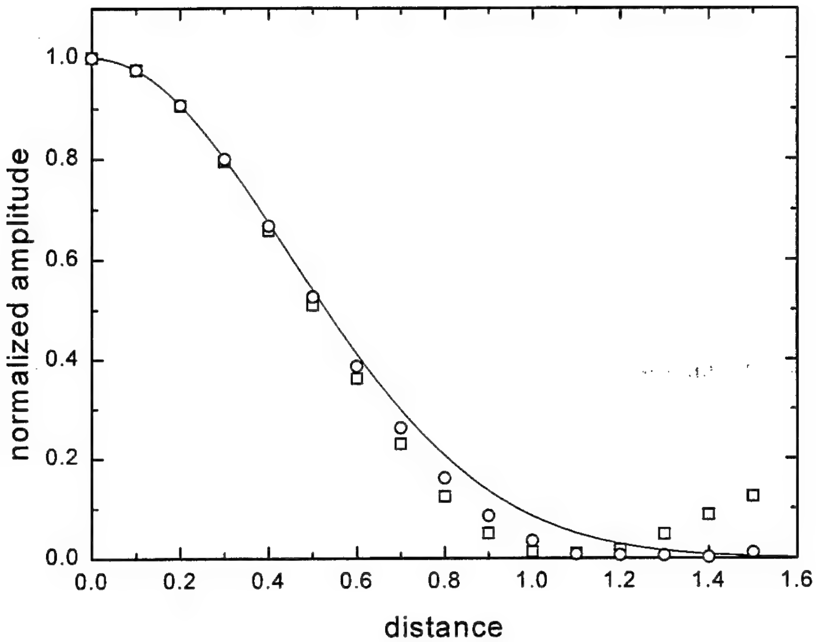

Fig 3.5 Attenuation of a signal with frequency 50 by three (squares) and four (circles) pumps. The rms particle velocity of the pumps is 10^{-2} and the frequencies are 1, $2^{1/4}$, $2^{1/3}$ for three pumps, and 1, $2^{1/4}$, $2^{1/3}$, $2^{1/2}$ for four pumps. The signal's amplitude is normalized to its value of 10^{-4} at the origin. The points are the result of numerical integration, and the solid line corresponds to the Gaussian attenuation law (2.13). The number of cycles of the signal is 10^{6} in the averaging.

IV. EXPERIMENT

A. APPARATUS

As stated in the Introduction, the experiment was conducted in a waveguide at frequencies below the first cutoff frequency to ensure one-dimensional propagation. Air was chosen as the acoustic medium. Other gases serve no justifiable advantage, and liquids are undesirable because their typical acoustic impedance is not substantially less that that of the confining waveguide material. (Such an acoustic impedance mismatch is required if plane-wave propagation is to be restricted to the medium that fills a waveguide.)

Fig. 4.1 shows a schematic diagram of the traveling wave tube built to perform the experiment and initially reported by Dorff (1991) . The tube includes seven 10-foot (3.05 m) sections of aluminum pipe with wall thickness 1/4 in. (6 mm) and inner diameter 1.96 in. (4.98 cm). For this diameter the first (azimuthal) cutoff frequency is 4.04 kHz, below which only plane waves can propagate. At one end are two compression drivers (J. B. Lansing model 2450H) connected to the tube by a "T" adapter, and at the other end is an anechoic termination.

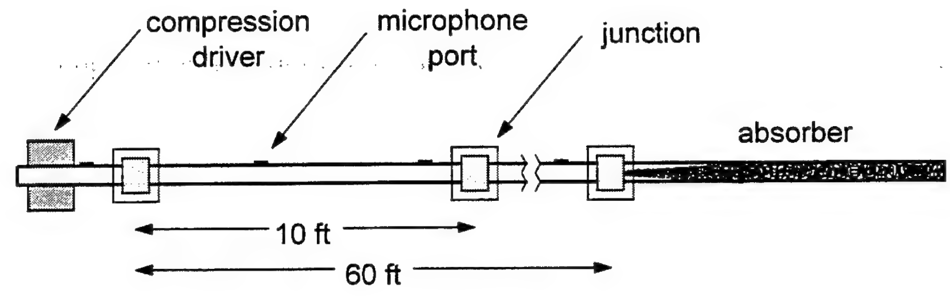

Figure 4.1 Traveling wave apparatus to observe the absorption of a monofrequency signal by nonlinear noise. One driver generates the signal, and the other the noise. The cross section of the tube is circular, with inner diameter 2.0 in. Both the signal and noise are plane waves.

To minimize reflections from the junctions and to reduce possible turbulence due to finite-amplitude effects, the sections of the pipe were smoothly joined as follows. The surfaces were only approximately circular and concentric, so the outer surface of each end was machined to a common diameter, and the inner surface was then machined with a slight (1°) taper opening to a common diameter at the end. The sections are connected with collars which provide alignment, and pairs of flanges which apply pressure between the contacting ends. Pulse measurements show that the reflections from the junctions are down at least 40 dB (1% in pressure deviation) for frequencies below the cutoff frequency of 4.0 kHz (Dorff, 1991).

Each of the first six sections of the tube has two threaded microphone ports with a uniform spacing of 5.00 ft. (1.52 m). An additional port exists 3.5 in. (8.9 cm) from the cavity formed by the drivers, and 4.50 ft. (1.37 m) from the first port of the first section. Data from a microphone at the cavity is not useful because standing waves prevent an accurate measurement of traveling wave amplitudes. Each port accommodates a piezoresistive pressure transducer (Endevco model 8510B) with a 10-32 mounting thread and a 0.15 inch (3.8 mm) diameter end that is flush along the inner wall of the tube. We used two such matched microphones in the experiment: one remained at the port nearest the drivers, and the other was moved from port to port as we gathered data. Each port not in use was closed with a screw that was machined to act as a substitute for the microphone.

The anechoic termination spans the length of the last section of the tube. The termination consists of 0000-grade steel wool with a tapered linear density that increases from zero to a maximum value in a manner according to Burns (1971), and is thereafter uniform. The taper occupies 1.0 m of the section; the remaining 2.0 m is uniform. The taper was achieved by first piecing together

pads of the steel wool to make a large bed, which were then cut according to the desired linear density and then rolled into a cylinder. The cylinder was fit into the tube by temporarily winding it with string and applying tension. As recorded by Dorff (1991), pulse measurements yield reflected waves that are at least 40 dB down for frequencies of 500 Hz and greater.

Tests on sum and difference generation of two signals reveal that the nonlinear interactions occur in the medium of interest (air), and are not the result of intermodulations of the drivers (Dorff, 1991). This indicates that the apparatus is suitable for an experiment to observe the absorption of sound by noise.

In the experiment, one driver was used to generate discrete "noise" of up to four spectral components (referred as pumps), and the other a monofrequency signal. The pumps were generated by an HP 8904 multifunction synthesizer that can provide up to four outputs of different frequencies, amplitudes, and relative phases. The power spectral density of the pumps was measured with a signal analyzer (HP 3561A). The frequency of the signal was typically 3.5 kHz. Due to both the cascading of the noise to higher frequencies and the attenuation of the signal, nonnegligible random variations of the signal occurred. Hence, to accurately extract the amplitude of the signal from the noise, we used a digital lock-in amplifier (SRS 850). The digital lock-in amplifier was set with a filter of 24 dB/octave and, for unequally spaced frequencies, a time constant of 30 s. Typically, after ten time constants a stable amplitude can be recorded. Thus for a 30 s time constant, the averaging time is 5 min, which for a frequency of 3.5 kHz, corresponds to 10^6 cycles. Not coincidentally, numerically we also required to average over 10^6 cycles of the signal for an stable output. Thus, a digital lock-in amplifier and a digital computer agree on the requisite number of cycles for a stable average.

B. MEASUREMENTS

We first established the maximum intensity of the discrete noise within the waveguide before shocks were formed. We achieved this by examining a single pump generated by the HP 8904. By adjusting the amplitude intensity of the pump, and monitoring a microphone output on an oscilloscope, discontinuities in the waveform were observed. The waveform was sampled at the final port, just before the anechoic termination of the waveguide. An input voltage was selected that did not cause the discontinuous waveforms at the final port; thus, the noise is ensured to be shockless over the entire length of waveguide.

For multiple pumps of different frequency, the input voltage of each component was determined by taking the maximum input voltage as described above, and dividing it by $\sqrt{N}$, where N is the number of pumps employed to characterize the noise. In this way the total intensity of the noise is constant. The importance of maintaining the same noise intensity is to be able to accurately compare the effects on the signal due to different number of noise components. The various input voltage used in the measurements are listed in Table 4.1.

Number of Components	1	2	3	4
Input Drive Voltage	165 mV	117 mV	96 mV	83 mV
Noise Spectral Intensity	480.5 mV	481.0 mV	479.8 mV	479.6 mV
Noise Spectral Level	142.45 dB	142.46 dB	142.44 dB	142.43 dB

Table 4.1 Input drive voltages from the HP8904 for various number of pumps used in experimental measurements.

The excess attenuation of the signal due to the nonlinear interactions with one, two, three, and four pumps was investigated. The frequencies of the pumps were selected within the band 0.5 to 1.0 kHz. As it will become clear later, the selection of this band was made because of the need to compare with earlier measurements of broadband noise with the same band, conducted by Larraza et al. (1996).

Figure 4.2 shows the attenuation of 3.5 kHz signal by one ($f_1 = 750$ Hz), two ($f_1 = 633$ Hz and $f_2 = 814$Hz), three ($f_1 = 633.3$ Hz, $f_2 = 762.5$ Hz, $f_3 = 893.1$ Hz,), and four ($f_1 = 533$ Hz, $f_2 = 676$ Hz, $f_3 = 780$ Hz, and $f_4 = 893$ Hz) pumps as a function of distance. The signal amplitude is the ratio of amplitude in the presence of the pumps to that of the amplitude without any pump, thus removing effects of wall losses on the signal.

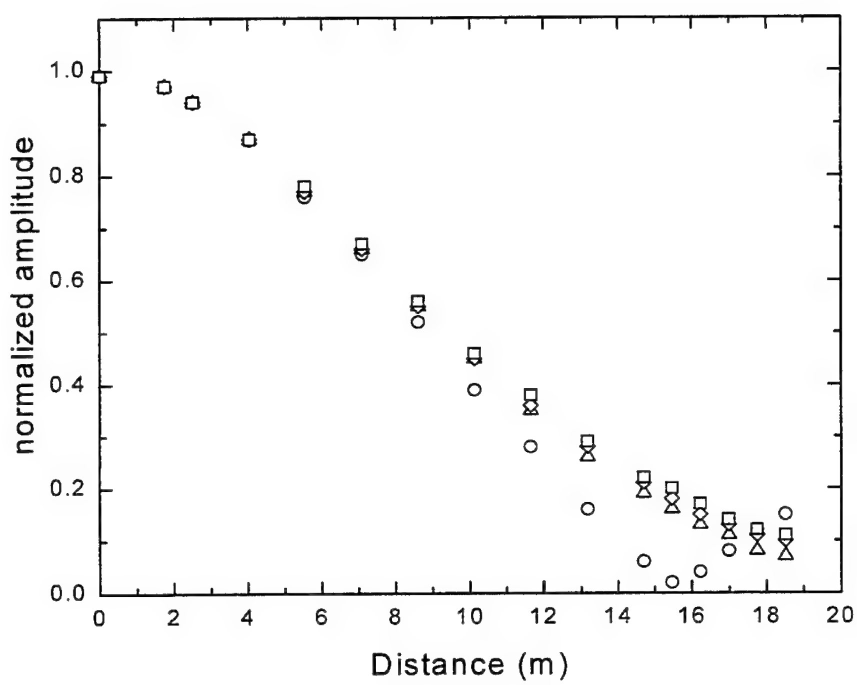

Figure 4.2 Attenuation of 3.5 kHz signal by 1(open circles), 2 (open triangles), 3 (open diamonds), and 4 (open squares) pumps as function of distance. The data corresponds to the ratio of the amplitude in the presence of pumps to the amplitude without any pump.

In agreement with the numerical results of the previous chapter, for unequally spaced frequencies of the pumps, we observed that the attenuation of the signal is independent of the relative phases of the pumps. However, with equally spaced frequencies the attenuation of the 3.5 kHz signal has a strong dependence upon the relative phases, also in agreement with the numerical results. We verified this statistically with different sets of random phases for both three and four pumps. Figures 4.3 and 4.4 shows histograms of the amplitude of a 3.5 kHz signal at a distance of 11.7 m from the source due to interactions with three and four pumps, respectively, where the frequencies of the pumps were equally spaced. The statistical samples correspond to fifty randomly selected sets of relative phases of the pumps. The mean amplitude of the signal at the location is 1.0786 mV for three pumps, and 1.266 mV for four pumps.

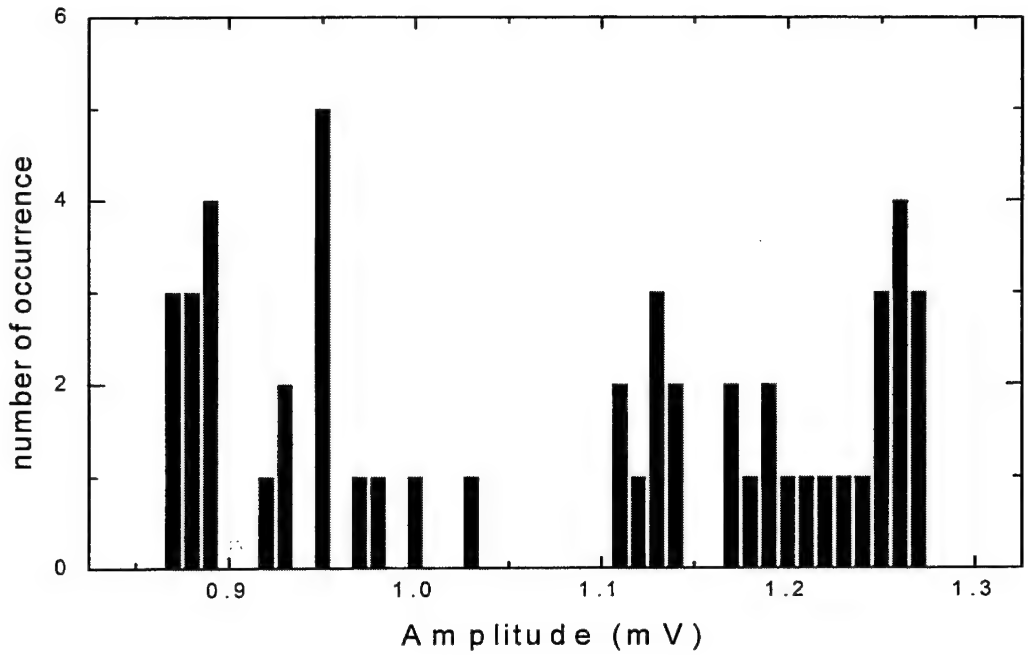

Figure 4.3 Histogram of the amplitude of the signal at 11.7 m from the source for 50 sets of relative phases of 3 pumps with equally spaced frequencies: 633 Hz, 763 Hz, and 893 Hz. The mean value of signal's amplitude is 1.0786 mV.

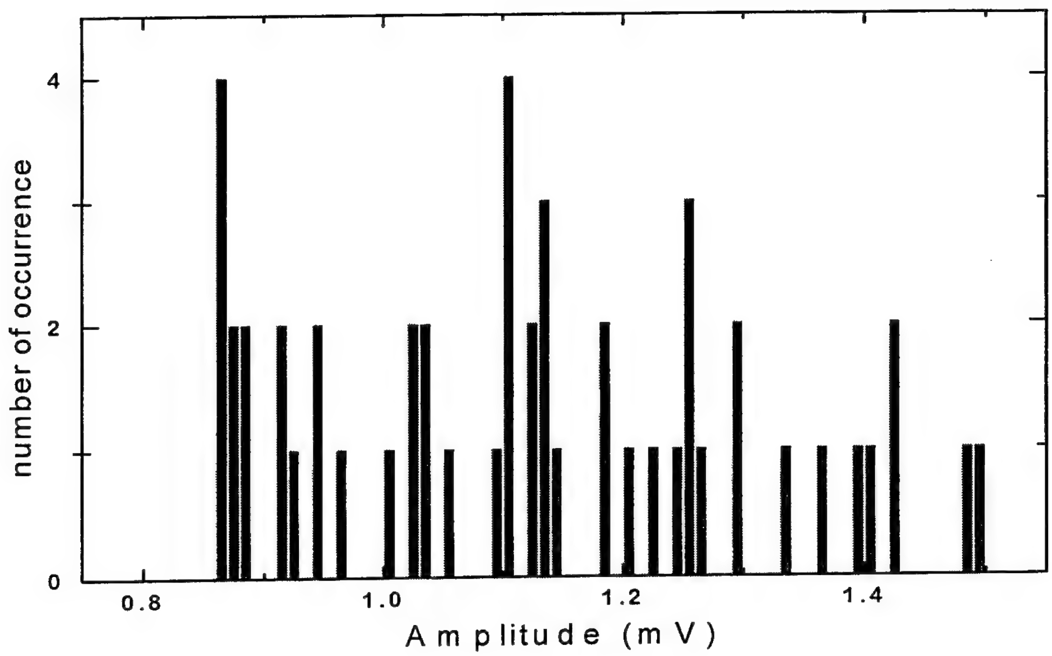

Figure 4.4 Histogram of the amplitude of the signal at 11.7 m from the source for 50 sets of relative phases of four pumps with equally spaced frequencies: 530 Hz, 675 Hz, 820 Hz, and 965 Hz. The mean value of signal's amplitude is 1.266 mV

We then randomly "tweaked" the equally spaced pumps by a tenth of a Hz, causing the spectral components to become unequally spaced. In this case, the amplitude of the signal at the same location of 11.7 m did not change for different values of the relative phases. Therefore, the excess attenuation of a signal is independent of the variation of the relative phases of the pumps when the spectral components are unequally spaced. Moreover, the measured value of the amplitude of the signal in the presence of three pumps was 1.07 mV while it was 1.13 mV for the case of four pumps.

Due to wall losses both the signal and the pumps undergo attenuation. A quantitative comparison between a theory and experiment requires that the wall losses be taken into account. Figure 4.5 show measurements of the amplitude

of the signal due to wall losses. The attenuation constant is obtained by a linear fit to the data and yields the value 1.256×10^{-3} Np/(m$\sqrt{\text{Hz}}$). This experimentally determined value is 7% greater than the theoretical value predicted for our apparatus. This difference is mainly a result of the tube not being perfectly rigid.

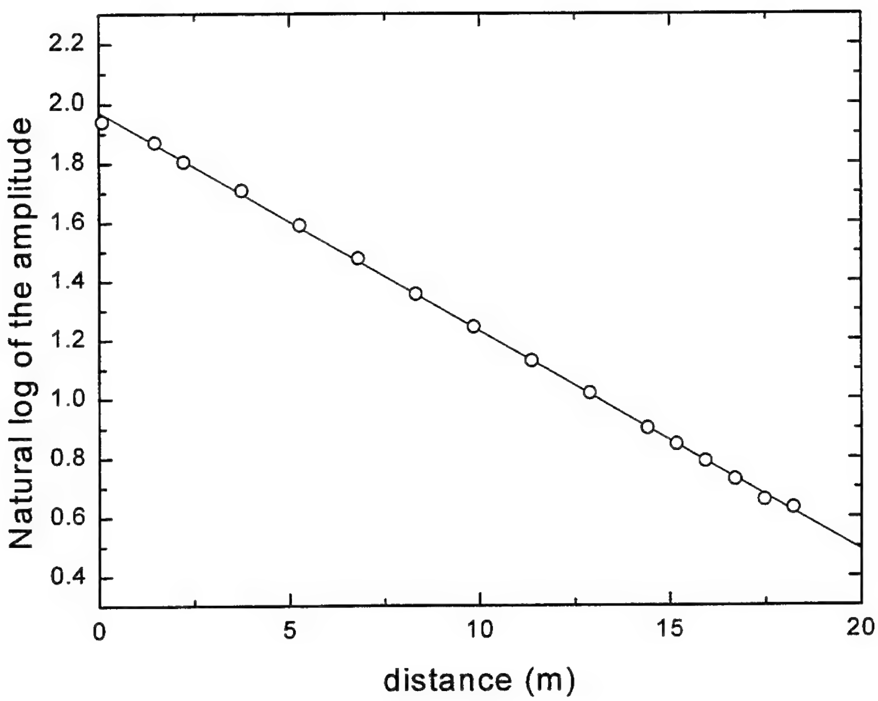

Figure 4.5 Attenuation of a 3.5 kHz signal due to wall losses. The attenuation constant $\alpha=1.256\times 10^{-3}$ Np/(m$\sqrt{\text{Hz}}$) was determined from a linear fit to the data.

Figure 4.6 shows the comparison between the theory, Eqs. (2.17) and (2.18), and experiment for the case of a single pump. The differential equation (2.17) was numerically solved (Appendix C) with boundary conditions A=1 and dA/dx=0, at x=0, and using the experimental parameters f_s =3.5 kHz for the signal, f_p =750 Hz for the pump, $u_p(0)$ = 0.64 m/s, and the empirical attenuation coefficient α = 0.001256 Np/(m$\sqrt{\text{Hz}}$).

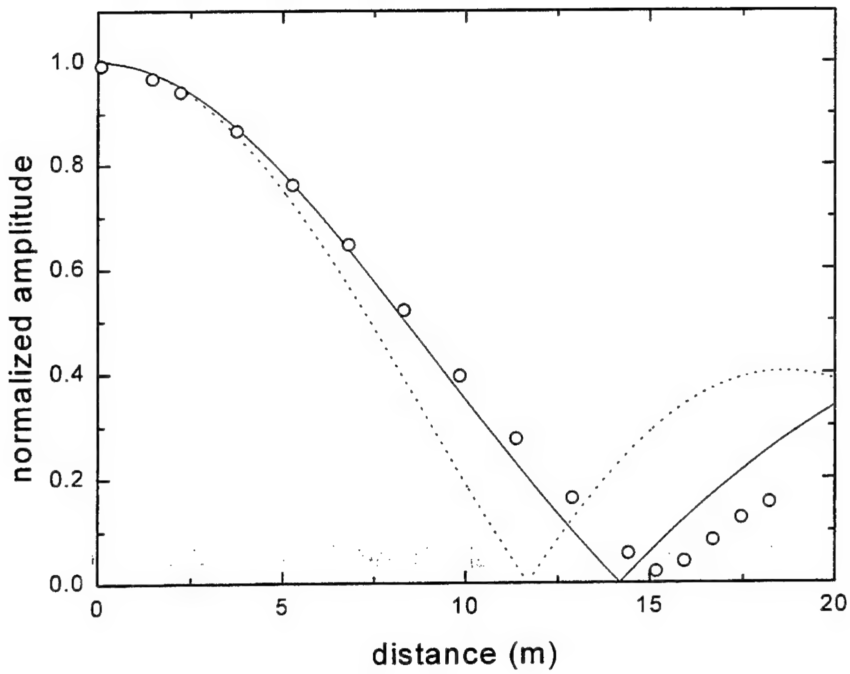

Figure 4.6 Excess attenuation a 3.5 kHz signal as a function of distance due to the nonlinear interaction with one pump. The solid line corresponds to theory, Eqs. (2.17) and (2.18), with no adjustable parameters. The dashed line is the theory in the absence of wall losses.

The theoretical curve shows reasonable agreement with experiment. The deviations can be understood by noting that the length scale of amplitude variations of the pump $1/\alpha\sqrt{f_p} \approx 29$ m, while the length scale of amplitude variations of the signal due to the pump is $c^2/\beta u_p \omega_s \approx 7$ m or 25% of the length scale of the pump's amplitude variations. Thus, the assumptions for a multiple length scale approach may be stretched in this case.

In the case of four pumps, we can compare the excess attenuation of the signal to that of earlier measurements of broadband noise. The broadband noise in Larraza et al. (1996) experiment was within a bandwidth of 0.5 to 1.0 kHz and had a maximum noise spectral level of 141 dB. The four discrete noise components used in our experiment were all within the same bandwidth, were not equally spaced, and had a noise spectral level of 143 dB. Figure 4.7 shows

the noise spectra for the two cases. In both cases, the spectra is measured at the first microphone port, which is 8.9 cm from the drivers.

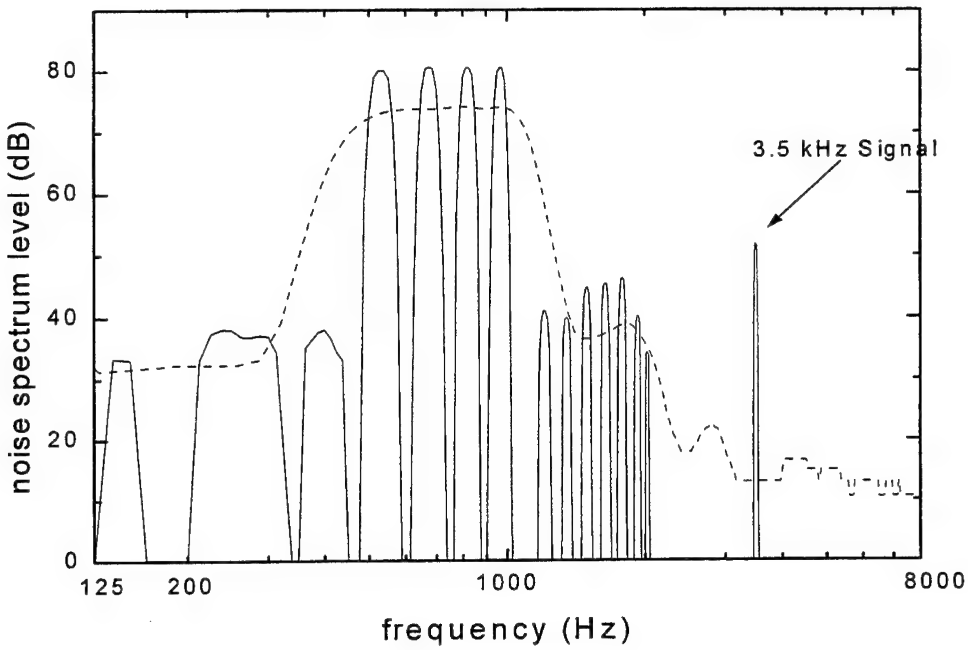

Figure 4.7 Comparison of the noise spectrum level (dB *re* 20 μPa / $\sqrt{Hz}$) of broadband shockless noise levels at the first microphone port, which is 8.9 cm from the drivers, to that of the spectrum level of four pumps. The 3.5 kHz signal is also displayed for reference.

Figure 4.8 shows the excess attenuation of the signal for the two cases. It should be noted that, due to the slightly higher noise level of the four-component case, the signal is expected to exhibit more attenuation. The general observation is that four discrete components roughly yield the same nonlinear attenuation as that of the broadband noise.

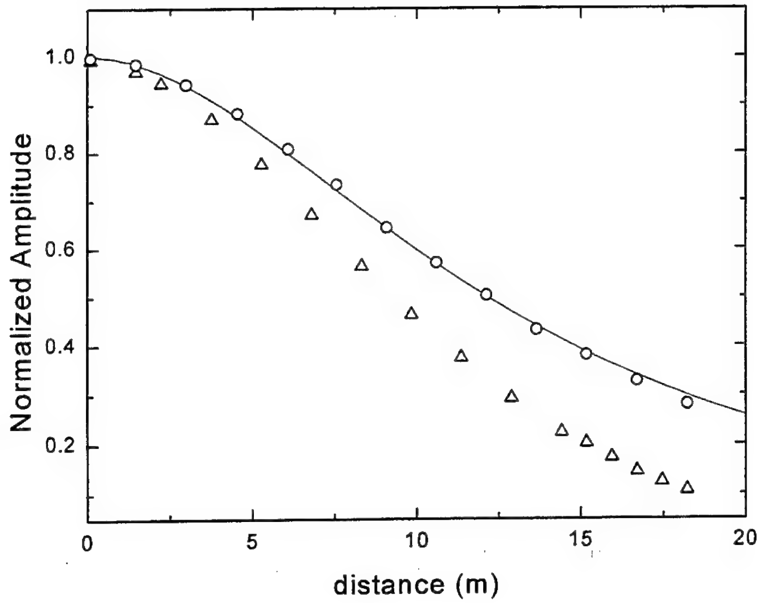

Figure 4.8 Excess attenuation of a 3.5 kHz signal due to nonlinear interactions with broadband noise (open circles) and with four pumps (open up triangles). The noise band is within 0.5 to 1.0 kHz with an intensity level of 141 dB. The four pumps are within the same bandwidth but with frequencies not equally spaced and an intensity of 143 dB. The data corresponds to the ratio of the amplitude in the presence of the noise or pumps to the amplitude without noise or pumps. The curve corresponds to theory, Eq. (2.16), with no adjustable parameters.

For noise with a discrete number of components, the solution to Eqs. (2.15) and (2.16) is

$$\frac{A(x)}{A_o} = \exp\left[\frac{-\pi^2\beta^2 f_s^2 u_{rms}^2}{c^4}\sum_{i=1}^{N}\left(\frac{1}{4\alpha^2 f_i}\left\{-2\alpha\sqrt{f_i}\,x e^{-2\alpha\sqrt{f_i}\,x} - e^{-2\alpha\sqrt{f_i}\,x} + 1\right\}\right)\right]. \quad (4.1)$$

If we specialize Eq (4.1) to the cases where there are only three and four noise components, we obtain the theoretical curves shown in Fig. 4.9. The experimental parameters were $u_{rms} = 0.6408$ m/s for the rms amplitude of a noise component, $\alpha = 1.2561\text{x}10^{-3}$ Np/(m√Hz), $f_s = 3.5$ kHz for the frequency of the signal. For the 3 pump case, the following frequencies were used f_1 =633.3 Hz,

$f_2 = 762.5$ Hz, $f_3 = 893.1$ Hz, and $f_1 = 533$ Hz, $f_2 = 676$ Hz, $f_3 = 780$ Hz, and $f_4 = 893$ Hz for the frequencies of the four unequally spaced pumps. There is a surprisingly very good agreement between experiment and the predicted Gaussian attenuation for the case of four pumps, but the agreement is not as good for three pumps. This indicates that the four unequally spaced pumps may be the minimum number of noise components to achieve the "thermodynamic" limit of broad band noise.

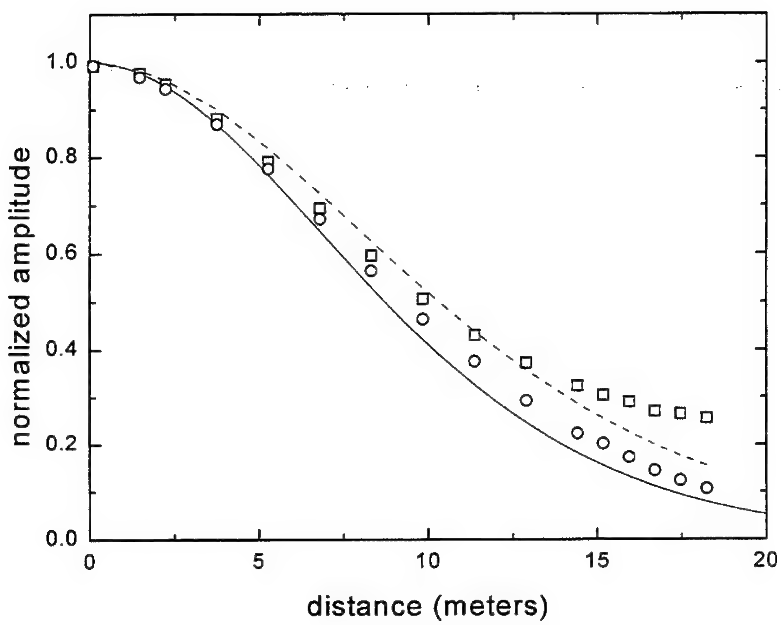

Figure 4.9 Excess attenuation of a signal in the presence of three (squares) and four (circles) pumps with unequally spaced frequencies. The curves are the theory Eq. (4.1) with no adjustable parameters: Dashed curve corresponds to the theory with three pumps, and the solid curve is the plot for the theory with four pumps.

We also conducted a preliminary experiment to examine the phase shift of a weak signal due to its interactions with 2, 3, and 4 pumps. The phase was measured at two locations, and phase differences were tabulated. The pumps had equally spaced frequencies and zero relative phases. The results of these measurements are shown in Table 4.2.

Number of components	Temp (°C)	Time	phase ϕ at port 0-B	phase ϕ at port 4-3	time constant τ	Remarks
4	20.5	1442 14Apr97	-124.55°	-101.63°	1 sec	without noise
4	20.5	1447 14Apr97	-124.55°	-97.54°	30 sec	with noise
4	20.5	1448 14Apr97	-124.54°	-98.88°	1 sec	without noise
3	20.8	1209 25Apr97	-124.48°	-68.39°	1 sec	without noise
3	20.8	1214 25Apr97	-124.46°	-66.20°	30 sec	with noise
3	20.8	1215 25Apr97	-124.48°	-69.02°	1 sec	without noise
2	20.8	1247 25Apr97	-125.48°	-72.40°	1 sec	without noise
2	20.8	1252 25Apr97	-125.37°	-70.00°	30 sec	with noise
2	20.8	1253 25Apr97	-125.47°	-72.99°	1 sec	without noise

Table 4.2 Measured phases of the 3.5 kHz signal in presence of various numbers of noise components. The microphone at port 4-3 is 11 m downstream form the microphone at the reference port 0-B located 0.87 m from the JBL drivers.

The phase recordings were taken from two separate SRS 850 digital lock-in amplifiers. The "sync" output of the HP 33120 A function generator used to drive the 3.5 kHz signal provided the reference frequency for both lock-in amplifiers. There was an observed drift in the phase over time, during what appeared to be constant conditions. An analysis of the drift did not appear to be related to the temperature variation during this period. To reduce the error due to drift, we recorded the pure tone signal phase before and after applying the noise components, and averaged the two values.

When only the signal is driven, the lock-in requires a small time constant, while a longer time constant is needed when the pumps are turned on. Taking the phase of the signal measured at the downstream port for 4, 3, and 2 pumps and subtracting the average phase of the signal in the absence of pumps, renders a positive phase shift of 2.715°, 2.505°, and 2.695°, respectively. Thus the phase of a weak signal becomes more positive in the presence of high intensity sound spectrum. In the presence of pumps, an upward phase shift of the signal indicates that the speed of sound increases with increasing intensity of the pumps.

V. CONCLUSIONS AND FUTURE WORK

A. CONCLUSIONS

We have experimentally and numerically investigated the attenuation of a monofrequency signal in the presence of a small discrete number of high intensity low frequency pumps in one dimension. Analogous to the fundamental question in the kinetic theory of gases regarding the minimum number of degrees of freedom that can represent the thermodynamic limit, we have probed the minimum number of spectral components that can represent noise. Because the interaction of a weak signal with broadband noise has been predicted (Rudenko and Chirkin, 1975) and experimentally measured (Larraza et al. 1996) to yield a Gaussian attenuation of the signal, we have used this attenuation law as our probe.

For equally spaced frequencies of the spectral components of the noise, the amplitude of a weak signal due to its interaction with the noise depends on the relative phase among the components of the noise. For unequally spaced frequencies, the signal's amplitude is independent of the relative phase. This behavior persists even for a larger number of discrete components. On the other hand, averaging different values of the signal amplitude corresponding to different realizations of the phases of equally spaced frequencies yields the same amplitude value as in the case of unequally spaced frequency components.

We observed, both numerically and experimentally, that at least four components would yield and attenuation law in close agreement with the Gaussian attenuation. This leads to the conclusion that the "thermodynamic" limit of nonlinear acoustic noise is nearly represented by four spectral

components with unequally spaced frequencies.

Preliminary observations of the change in phase of a weak signal in the presence of two, three, and four noise components reveal that the phase becomes more positive in the presence of high intensity sound spectrum. An upward phase shift of the signal indicates that the speed of sound increases with increasing intensity of the pumps. This constitutes the first observation of such phase shift. The significance of this result lies in the possibility of observing collective modes, in which energy fluctuations propagate (Larraza and Falkovich, 1993). Although such modes in far off equilibrium noise have been predicted in several systems, there are currently no controlled observations. These modes are important, for example, as a mechanism of energy transfer.

B. FUTURE WORK

Future investigations should test the statistics of the numerical noise and quantify the deviations from normal distribution at the origin. Deviations from the normal distribution should also be quantified downstream where strong correlations may develop due to nonlinearities. A comparison between equally and unequally spaced frequencies should be made, both at the origin and downstream. Due to nonlinearities the energy cascade to different frequencies may cause a significantly different evolution of the statistics of the noise between these two.

Because a substantially small frequency components of the noise are required to yield an approximate Gaussian attenuation of a signal, this may be important in applications where an efficient suppression of high frequency sound is desired.

Measurements of the phase shift of a weak signal are just the beginning of a search for collective modes. The phase shift should be measured as a function of the intensity, number of components, relative phases, frequency spacing, and also for broadband noise.

APPENDIX A. FORWARD TIME NUMERICAL PROGRAM THAT DETERMINES THE PROPER NUMBER OF ITERATION CYCLES FOR STABLE SIGNAL AMPLITUDE

```
/**********************************************************/
/*      Numerical integration of Riemann's equation      */
/*       for propagation of sound in one dimension       */
/*                File Name:  SIZE.C                     */
/**********************************************************/

/* This program simulates the propagation of a CW in the
   presence of one dimensional noise with a flat
   distribution.  The signal's amplitude is calculated as a
   function of distance.  The noise is discrete and the
   number of components N can be made to vary from one to
   several.  Regardless of the number of components, the
   total energy of the noise is kept constant by dividing
   the peak amplitude of each noise component by the
   square root of N.

   This modification utilizes randomly generated (control by
   varying seed values)numbers to initialize the phase and
   unequal spacing of noise components.  In doing so the
   proper number of averages or iteration cycles to yield a
   stable signal amplitude output can be determined.

    LAST UPDATE : 13 May 1997, by Mark Lamczyk */
```

```
/*****     PROCESSOR DIRECTIVES     *****/
#include "math.h"
#include "stdio.h"
#include "stdlib.h"

/*****     MACRO DEFINITIONS     *****/
#define pi 3.141592654
#define fs 50.0       /* frequency of signal */
#define us 0.0001     /* dimensionless peak velocity of one of
                         noise components */

/*****     GLOBAL VARIABLES     *****/
double x, u0,         /* distance */
       time,          /* time  */
       r[100],        /* random phased */
       rr[100],       /* random nonperiodic term */
       velocity;      /* dimensionless particle velocity */
int N;

/*****     SUB ROUTINES     *****/
double retard(void), initial_velocity(void);

void main()
{   /*** begin main ***/
  int i;j,k,SIZE, seed;

  double R[100],
```

```
        signal,             /* extracted signal component
                               from noise */
        old_time,new_time,/* retarded time */
        dx,                 /* distance step */
        t,dt,               /* time and time step */
        random,             /* seed for random number
                               generation*/
        a1,a2,d1,d2;        /* variables used for LOCK-IN */

FILE *fp;                   /* initiates the creation of
                               output data file */

/*****     Input Parameters     *****/
printf("\n Enter the number for the seed:");
scanf("%d", &seed);
printf("\nThe seed is %d \n", seed);

printf("\n Enter the number of Noise components (N>0):");
scanf("%d", &N);
printf("\nN is %d \n", N);

u0=0.01*sqrt(1.0/N); /* dimensionless peak velocity of one
                        of noise components/
                     /* thus, the noise energy is the
                        input u0^2 */

printf("\n Enter a value for SIZE:");
```

```
scanf("%d", &SIZE);
printf("\nSIZE is %d \n", SIZE);

/*****     Variables Initialization     *****/
d1=d2=t=x=0.0;
dt=0.001;
dx=0.1;

for(i=0; i<100; i++){
   R[i]=0.0; }

for(j=0; j<1; j++){  /*** begin j loop ***/
   srand(seed);

   for(i=0; i<N; i++){ /**begin random generation loop**/
      r[i]=(2.0*pi*rand()/RAND_MAX); /* Random phase of
                                        noise components */
      rr[i]=(rand()/RAND_MAX)/100.0;  /* Random spacing of
                                         noise components */
      printf("%d %.12f     %.12f\n",i, r[i], rr[i]);
      } /*** end random generation loop ***/

      for(k=1; k<21; k++) {  /***  begin k loop  ***/
      time=0.0;
      initial_velocity();
      retard();
      old_time=time;
```

```
for(i=0; i<SIZE; i++){  /*** begin i loop ***/
   signal=us*sin(2.0*pi*fs*old_time);  /* signal
                                           only */
   time=(i+1)*dt;
   initial_velocity();
   retard();
   new_time=time;

   t=i*dt;
   a1=signal*cos(2.0*pi*fs*t);
   a2=signal*sin(2.0*pi*fs*t);
   d1=d1+a1*(new_time-old_time);
   d2=d2+a2*(new_time-old_time);
   old_time=new_time;

}  /*** end i loop ***/

R[k]=R[k]+(2.0*fs*sqrt(d1*d1+d2*d2)/(fs*dt*SIZE*us));
x=k*dx;
d1=d2=t=0.0;

} /***  end k loop  ***/

seed=seed+1.0;
x=0.0;
}  /*** end j loop ***/
```

```
 x=0.0;
 /*****     Print the result     *****/
 for(i=1; i<21; i++) {  /***  begin 2nd i loop  ***/
   x=(i-1)*dx;
   fp=fopen("thesis1.dat","a");  /* Open the output file
                                    named as thesis1.dat */
   fprintf(fp,"%.12f %d %d %d      %.12f\n",x, seed, N,
            SIZE,R[i]/1.0);
   fclose(fp);
   printf("%.12f %d %d %d     %.12f\n",x, seed, N,
            SIZE,R[i]/1.0);
  }  /*** end 2nd i loop  ***/

}  /*** end main ***/



/*************** sub routines ***************/
double initial_velocity()
{  /*** begin initial_velocity sub routine ***/
 int i;
 double vel,var;

 var=0.0;
 for(i=0; i<N; i++){ /*** begin i loop ***/
```

```
 /*
vel=u0*sin(2.0*pi*(1.0+(double)(i)/(double)(N))*time+r[i]);
      periodic noise condition */
/* NonPeriodic noise condition */
vel=u0*sin(2.0*pi*(1.0+(double)(i)/(double)(N)+rr[i])*time+r
[i]);

   var=var+vel;

   } /*** end i loop ***/

velocity=var+us*sin(2.0*pi*fs*time);
return velocity;
}  /*** end initial_velocity sub routine ***/


double retard()
{  /*** begin retard sub routine ***/
 time=time-x/(1.0+velocity);
 return time;
}  /*** end retard sub routine ***/
```

APPENDIX B. FORWARD TIME NUMERICAL PROGRAM FOR THE CASE OF UNEQUALLY SPACED NOISE PUMPS

```
/**********************************************************/
/*     Numerical integration of Riemann's equation      */
/*      for propagation of sound in one dimension       */
/*             File Name:  NONPERIODIC.C                */
/**********************************************************/

/* This program simulates the propagation of a CW in the
   presence of one dimensional noise with a flat
   distribution.  The signal's amplitude is calculated as a
   function of distance.  The noise is discrete and the
   number of components N can be made to vary from one to
   several.  Regardless of the number of components, the
   total energy of the noise is kept constant by dividing
   the peak amplitude of each noise component by the
   square root of N.

   This modification utilizes randomly generated (control by
   varying seed values)numbers to initialize the phase and
   unequal spacing of noise components. Fixing the
   frequencies an unequal values was also conducted by
   commenting out the random frequency spacing statement.
   The signal attenuation due to the nonlinear interaction
   with the nonperiodic noise components is determined at
   various downstream locations. The simulation model does
```

```
   not take in account the dissipation to wall losses.

    LAST UPDATE : 28 May 1997 by Mark Lamczyk */

/*****      PROCESSOR DIRECTIVES      *****/
#include "math.h"
#include "stdio.h"
#include "stdlib.h"

/*****      MACRO DEFINITIONS      *****/
#define pi 3.141592654
#define SIZE 1000000 /*numerically determine value that
                      yields a stable signal amplitude.
                      From THESIS.C */
#define fs 50.0     /*frequency of signal */
#define us 0.0001   /*dimensionless peak velocity of one of
                     noise Components */

/*****      GLOBAL VARIABLES      *****/
double x, u0,       /* distance */
       time,        /* time  */
       f[5],        /* fixed frequencies */
       r[100],      /* random phased */
       rr[100],     /* random nonperiodic term */
       velocity;    /* dimensionless particle velocity */
int N;
```

```
/*****      SUB ROUTINES      *****/
double retard(void),  initial_velocity(void);


void main()
{  /*** begin main ***/
  int     i,j,k;


  double R[100],
         signal,            /* extracted signal component
                               from noise */
         old_time,new_time,/* retarded time */
         dx,                /* distance step */
         t,dt,              /* time and time step */
         seed,              /* seed for random number
                               generation */
         a1,a2,d1,d2;       /* variables used for LOCK-IN */


  FILE *fp;
  fp=fopen("out2.dat","w"); /* Open the output file named as
                               out1.dat */


  /*****      Input Parameters      *****/
  printf("\n Enter the number of Noise components (N>0):");
  scanf("%d", &N);
  printf("\nN is %d \n", N);
  u0=0.01*sqrt(1.0/N);
```

```
seed=0.0;
d1=d2=t=x=0.0;
dt=0.001;
dx=0.1;

for(i=0; i<100; i++){
   R[i]=0.0;   }

   srand(seed);
   for(i=0; i<N; i++){ /** begin random generation loop */
      r[i]=(2.0*pi*rand()/RAND_MAX);   /* Random phase of
                                          noise components */
      /*  rr[i]=(rand()/RAND_MAX)/100.0; */
                      /* Random spacing of noise components */
      }  /*** end random generation loop ***/

   f[0]=1.0;   f[1]=1.189207115;
   f[2]=1.25992105;   f[3]=1.41413562;

   for(k=1; k<17; k++){   /*** begin k loop ***/
      time=0.0;
      initial_velocity();
      retard();
      old_time=time;

      for(i=0; i<SIZE; i++){ /*** begin i loop ***/
```

```
        signal=us*sin(2.0*pi*fs*old_time); /* signal
                                              only */
        time=(i+1)*dt;
        initial_velocity();
        retard();
        new_time=time;

        t=i*dt;
        a1=signal*cos(2.0*pi*fs*t);
        a2=signal*sin(2.0*pi*fs*t);
        d1=d1+a1*(new_time-old_time);
        d2=d2+a2*(new_time-old_time);
        old_time=new_time;

        } /*** end i loop ***/

      R[k]=R[k]+(2.0*fs*sqrt(d1*d1+d2*d2)/(fs*dt*SIZE*us));
      x=k*dx; d1=d2=t=0.0;

      } /*** end k loop ***/

  x=0.0;
  /*****     Print the result     *****/
  for(i=1; i<17; i++){ /* Begin second i loop */
    fprintf(fp,"%f       %.12f\n",x,R[i]/1.0);
    x=i*dx;
    } /*** end second i loop ***/
```

```
  fclose(fp);

} /*** end main ***/


/*************** sub routines ***************/

double initial_velocity()
{ /*** begin initial_velocity sub routine ***/
int i;
double vel,var;
var=0.0;

for(i=0; i<N; i++){ /* begin third i loop ***/

/* vel=u0*sin(2.0*pi*(1.0+(((double)(i)/(double)(N))+rr[i]))
     *time+r[i]); */  /* For Random Freq. Generation */

 vel=u0*sin((2.0*pi*(f[i])*time)+r[i]); /* Fixed Freqs */

  var=var+vel;
} /*** end third i loop ***/

velocity=var+us*sin(2.0*pi*fs*time);
return velocity;
} /*** end initial_velocity sub routine ***/
```

```
double retard()
{ /*** begin retard sub routine ***/
 time=time-x/(1.0+velocity);
 return time;
} /*** end retard sub routine ***/
```

APPENDIX C. NUMERICAL SOLUTION TO THE MODIFIED BESSEL DIFFERENTIAL EQUATION 2.17

```
/*********************************************************/
/* Thesis Simulation:  Numerical solution to modified    */
/* bessel differential equation.  For the case of Signal */
/* attenuation due to the nonlinear interaction with a   */
/* single noise pump.                                    */
/* Naval Postgraduate School, Monterey, California       */
/* File Name: bessel.c                                   */
/* Date last modified: 10 April 1997 by Mark A. Lamczyk  */
/*********************************************************/

/*****      PROCESSOR DIRECTIVES      *****/
#include "math.h"
#include "stdio.h"
#include "stdlib.h"

/*****      MACRO DEFINITIONS     *****/
#define pi 3.141592654

/*****      GLOBAL VARIABLES     *****/
float A, B, alpha, x, dx, xmax, amp, aa, ab, ALPHA;
float fp, fs, beta, c, C, ALPHAstar, Uo, signal;

main()
{ /*** begin main ***/
```

```
/*****      Experimental Constant     *****/
A=7.19;                /* boundary condition */
B=0.0;                 /* boundary condition */
dx=0.01; xmax=20;
c=341; beta=1.2;
fp=750; fs=3500;

/*****      Input Parameters      *****/
printf("Enter a value for alpha:");
scanf("%f", &alpha);
printf("\nalpha is %f", alpha);

printf("\n Enter a value for input noise intensity (
mVolts):");
scanf("%f", &signal);
printf("\nInput Noise intensity is %f mVolts \n", signal);

/*****      Convert Raw Experimental Constant into
Theoretical
            Terms    *****/
Uo=(signal/200.0)*(1.0/61.61)*(1.0/14.7)*(1.0e5/1.0)*(1.0/41
5.0);
ALPHAstar=-2.0*alpha*sqrt(fp);
ALPHA=alpha*sqrt(fs);

C=(4.0*(pi*pi)*(fs*fs)*(beta*beta)*2.0*(Uo*Uo))/(c*c*c*c);
```

```
for(x=0.1; x<xmax; x+=dx) { /*** begin x loop ***/
  /* A & B are the coupled terms of the differential
     equation */
  B=B-((1.0/x)*B+(C*exp(ALPHAstar*x))*A)*dx;
  A=A+(B*dx);
  aa=A*A;  ab=sqrt(aa);   /* taking absolute value of A */
  amp=ab/7.19;  /* removing the attenuation due to wall
                   losses in order to extract only the
                   excess attenuated signal due to the
                   presence of the noise */
  printf("%f \t%f\n",x,amp);
  } /*** end x loop ***/

} /*** end main ***/
```

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